Synchronised multi-perspective analysis of online mathematical argument

1 A multi-perspective approach

Polymath [20] is an online experiment in collaborative mathematics. Participants work together to solve either open problems in mathematics, or questions from the International Mathematical Olympiad (IMO). The work is carried out online using a threaded discussion system for research and a wiki to represent the current state of knowledge.

This provides a compelling test-bed for developing theories of argumentation, as there is *(i)* a discourse where utterances are structured so that it is clear which post any comment relates to; *(ii)* a domain of discourse which can be formally represented; *(iii)* a synchronised representation of both how the dialogue unfolds over time and common knowledge which is a result of the dialogue; *(iv)* the potential for mechanical assistance to be brought in to the interaction if the activity can be sufficiently represented in computational terms.

From an argumentation perspective, there are several aspects of this interaction that can be analysed under different paradigms. In order to develop a rich understanding of how the process of doing mathematics collaborative is carried out, we analyse the same set of traces from multiple perspectives:

- A Lakatosian viewpoint [5], examining the utterances which fit into the schema of proofs and refutations, examples, counterexamples. Here we look only at those contributions which are congruent with Lakatos theories of how maths is done. This is an attempt to fit an existing structure to an example of observed argumentation.
- A process calculus analysis [8, 14] which attempts to structure all of the utterances as a series of communications, using relations developed through analysis of the text. Here, we try to synthesis a protocol which can both account for the behaviour observed, and through its formality admits the possibility of computational support.
- Inference Anchoring Theory (IAT) [2] which is developed through the interaction, which links into the utterances that provide components of the proof structure. This shows how the object under discussion relates to the text, highlighting the gaps in the structure due to shared context and implicitly understood reasoning.

Finally, we integrate these analyses around a temporal axis, and compare with the proof structures created to understand how the dynamic, sequential process of argumentation, of example and counterexample come together to form a coherent argument. This allows us to place the alternative perspectives alongside each other to demonstrate correspondences, areas of overlap and points of disagreement.

2 Related Work

Stahl has done extensive research on mathematics in a social context [17, 18, 19], developing research into "*group cognition*," drawing on online interactions between students. His notion of "*adjacency pairs*" is a broader category than what we call Lakatosian moves. Stahl's work focuses on computer supported collaborative learning, which is an inspiring domain for our work, although we also aim to add dimensions related to computer *simulated* collaborative learning. Nevertheless, our approach in the current phase of work, in which we will closely examine real-world dialogues, is similar to Stahl's. In our preliminary work, we have sought to establish a process coding suited to the social mathematics context. Earlier research focused on tagging texts created through speak-aloud protocals, for instance, Lucas et al. [7]. More recent work by Schoenfeld [15] situates mathematical problem solving in a social context, but the analysis of this material generally excludes social and cognitive details [15, p. 16]. Contemporary strategies from natural language processing, including the field of *argument mining* [11] which has gained recently traction within the broader field of *discourse mining* (e.g. [16, 21]) are also closely related, although here we only developing groundwork for future NLP-based efforts. In the current work we will not only look at argumentative structures, but also at pre-argumentative structures, that is, we consider the constructive features of *informal logic* (cf. [4]).

3 Illustrative Ideas

In Figures 1 and 2, we present short excerpts from MiniPolymath1 and MiniPolymath3, respectively, coded with tags drawn from the frameworks collected in Table 1. The tag sets `SA`, `AD`, `PD` are connected with primarily *dialogical* features; `MC`, `MO`, and `LD` with *logical* features; and `SS`, `PP`, `SF`, and `CE`, with *pragmatic* features. The first excerpt is adequately described as classic problem solving; it includes 6 "pragmatic" moves and 1 "dialogical" move. By contrast, the second is much more discursive, and includes 2 "pragmatic" moves, 3 "logical" moves, and 1 "dialogical" move. These distinctions are somewhat ad hoc; what is important to note is that earlier research has focused primarily on ("pragmatic") problem solving. When compared with the 42 primary tags used in [7], the tags that we used are much less focused on problem solving *per se*, although there is a significant overlap in the terminology and sources used. Our tag set will need to expand further in order to deal adequately with issues related to problem identification, positing, and selection.

Nate: Well, my first thought is to see if the hypotheses seem reasonable.[ss3] The hypothesis that $s = a_1 + \ldots + a_n$ not lie in M is certainly necessary, as the last jump that the grasshopper takes will land on s.[pp2] The grasshopper's other steps will land on a partial sums $a_{\sigma(1)} + \ldots + a_{\sigma(k)}$ for some permutation σ, but we get to choose the permutation. Thus it seems plausible that we can avoid a given set of $n-1$ points.[ss2]

Thomas: Quick observation.[sa2] The grasshopper must make a first step.[sf2] This is always possible, since the a_i are distinct and $|M| = n-1$; that is, there is always an a_i not in M.[pp2] However, let's say M matches all but one of the a_i. Then the first step is uniquely determined. Still, according to the claimed theorem, a second step must still be possible.[pp3]

[ss3] explore
[pp2] conditions
[ss2] analyze
[sa2] inform
[sf2] heuristic (simplify)
[pp2] conditions
[pp3] decomposition

Figure 1: Excerpt from MiniPolymath1

Haggai Nuchi: The first point and line P_0, l_0 cannot be chosen[ce7] so that P_0 is on the boundary of the convex hull of S and l_0 picks out an adjacent point on the convex hull.[mc2] Maybe the strategy should be to take out the convex hull of S from consideration; follow it up by induction on removing successive convex hulls.[ld7]

Haggai Nuchi: More specifically, remove the subset of S which forms the convex hull to get S_1; remove the new convex hull to get S_2, and repeat until S_n is convex. Maybe a point of S_n is a good place to start.[ss5]

Srivatsan Narayanan: Can we just assume by induction that we have proved the result for all the "inner points" $S_2 \cup S_3 \cdot \cup S_n$.[ad5] The base case would be that $S = S_1$, i.e., it forms a convex polygon.[mc4]

[ce7] assertion
[mc2] example (monster)
[ld7] lemma incorporation
[ss5] implement
[ad5] negotiation
[mc4] proof

Figure 2: Excerpt from MiniPolymath3

SA	ackn, inform	*Speech-Act Annotated Corpus Classified List of Speech Acts* [6]
AD	negotiation	*Aberdein's patterns of proof dialogue* [1]
PD	question	*Prakken's persuasion dialogues* [13]
LD	conjecture, lemma incorporation	*Lakatosian moves* [9]
MC	concept, example (arbitrary inst.), example (monster), proof, other (phatic)	*types of mathematical comments* [10]
MO	article, problem, ephemera	*types of mathematical objects* [3]
SS	read, analyze, explore, implement, verify, local assessments, transition	*Schoenfeld's interpretation of "How to Solve It"* [15]
PP	goal, conditions, decomposition	*Pólya's stages of planning* [12]
SF	resources, heuristic (compute!), heuristic (decompose), heuristic (formal gen.), heuristic (simplify), heuristic (symmetry), heuristic (total stuckness), control, belief systems	*Schoenfeld's factors in mathematical thinking* [15]
CE	property, assertion	*components of explanation* [in preparation]

Table 1: Relevant tag sets with example tags

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